

Ma 4260
Sample first exam
Su 98

Name _____

You may use your calculator, your crib sheet, and the tables supplied. Please use no other outside source of information. **Show your work.**

1. Suppose X_1, \dots, X_n are, for fixed, i. i. d. with probability density

$$f(x | \theta) = \begin{cases} e^{-x/\theta} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}.$$

1a) Give the joint probability density of the X 's given θ .

1b) Give the Bayes estimator of θ under squared error loss if the prior on θ is the gamma distribution $G(\alpha, \beta)$.

2. Consider the random variable Z whose probability density is

$$f(z) = \begin{cases} \frac{1}{\Gamma(\alpha)} e^{-z/\alpha} z^{\alpha-1} & \text{if } z > 0 \\ 0 & \text{otherwise} \end{cases}.$$

2a) Find the mode (location of the maximum) of this density if $\alpha > 1$.

2b) Show that $E(e^{-z/\alpha}) = \frac{\Gamma(\alpha-1)}{\Gamma(\alpha)}$ if $\alpha > 1$ is any real number such that $\alpha > 1$. (Hint : For every $u > 0$

and $v > 0$ the function of u given by $f(u) = \begin{cases} \frac{u-1}{\Gamma(u)} e^{-u/v} & \text{if } u > 0 \\ 0 & \text{otherwise} \end{cases}$ is a probability density.)

3. If X_1, \dots, X_n are i. i. d. with probability density

$$f(x | \theta) = \begin{cases} e^{-x/\theta} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

3a) Find the maximum likelihood estimator $\hat{\theta}$ of θ .

3b) What is the probability distribution of $\sum_{i=1}^n X_i$?

3c) Is $\hat{\theta}$ is an unbiased estimator of θ ? (Hint : Use the result of part b of problem 2) If it is not, give an unbiased estimator $\tilde{\theta}$ of θ .

3d) State a relationship between maximum likelihood estimators and efficient estimators.

3e) Is $\tilde{\theta}$ efficient?

4. Poisoning by DDT causes tremors and convulsions. In a study of DDT poisoning, four rats randomly selected from a certain strain were fed a measured amount of DDT and their absolute refractory periods were measured (the absolute refractory period is the time required for a nerve to recover after a stimulus). Assuming the absolute refractory period is normally distributed, give a 90% confidence interval for the mean absolute refractory period for the rats of the strain from which the following four measurements were taken.

1.25 1.43 1.35 1.52

5. An entomologist samples a field for egg masses of a harmful insect by placing a yard-square frame at random locations and carefully examining the ground within the frame. A random sample of 50 locations selected from a county's pasture land found egg masses in 13 locations. Give a 95% confidence interval for the proportion of of all possible locations that are infested.