Ma 4260 Exam II Sample Name________________________

Table #

You may use your calculator, your crib sheets, and the tables supplied. Please use no other outside source of information. **Show your work**.

1. $X_1, \ldots, X_n$ are iid $N(\mu, \sigma^2)$ and σ is known.

(a) The joint density of the X's is $f(x_1, \ldots, x_n \mid \mu) =$

(b) Derive the maximum likelihood estimator $\hat{\mu}$ of μ.

(c) The likelihood ratio for the hypothesis

(*) $H_0 : \mu = 0$ against $H_A : \mu \neq 0$

is $\lambda(x) =$

(d) Prove that $\sum_{j=1}^{n} (x_j - t)^2 = \sum_{j=1}^{n} (x_j - \bar{x})^2 + n(\bar{x} - t)^2$ and use this to prove that $\lambda(x) \leq c$ if and only if $|\bar{x}|$ is too large.

(e) Show that the likelihood ratio test of (*) of size α rejects H_0 if $\sqrt{n}\,|\bar{x}|/\sigma > z_{\alpha/2}$.

2. Assuming the data below constitutes a random sample from a normal distribution, does the data present significant evidence at $\alpha = .05$ that the mean of the population differs from 7? You may use, without derivation, the likelihood ratio test we developed in class for $H_0: \mu = \mu_0$ against $H_A: \mu \neq \mu_0$. Be sure to state the test before you use it.

1.92 3.84 5.50 2.62

3. If $x_1, \ldots, x_n$ are fixed numbers, not all zero, and if $Y_1, \ldots, Y_n$ are independent,

$$Y_i \sim N(\theta x_i, 1)$$

then

a) the joint density of the Y's is

$f(y_1, \ldots, y_n \mid \theta) =$

b) Give the form of the Neyman-Pearson most powerful test of

$$H_0: \theta = 0 \quad \text{against} \quad H_A: \theta = 1$$

c) What is the probability distribution of $a_1 Y_1 + \ldots + a_n Y_n$?

d) Give the Neyman-Pearson most powerful test of the hypotheses in (b) of size $\alpha = 0.05$.

e) Is the test in d) UMP (uniformly most powerful) of size $\alpha = 0.05$ for testing

$$H_0: \theta = 0 \quad \text{against} \quad H_A: \theta > 0 \text{ ?}$$

Why?