

## The F distribution.

This distribution is named in honor of R. A. Fisher. Aside from the information we developed in class, the following is crucial in being able to utilize the tables. In making tests and setting confidence intervals we require the cutoffs  $F_{r,s}$  and  $F_{r,s,1-\alpha}$ , where typically  $\alpha$  (0,1) is small (so  $1-\alpha$  is large). Consulting tables one finds that the former is there, but not the latter, at least not directly. The fact is that the tables can be used for both. The key is that

$$(*) \quad F_{r,s,1-\alpha} = 1 / F_{s,r, \alpha}$$

so one simply reverses the degrees of freedom and takes one over the value at the upper cutoff. The reason this works is proven next

. Recall that, by definition, the  $F_{r,s}$  distribution is the probability distribution of the ratio  $Y = \frac{U/r}{V/s}$ , where  $U \sim \chi^2_r$  and  $V \sim \chi^2_s$  are independent chi-square random variables.

The proof of (\*) is that

$$P[Y > F_{r,s,1-\alpha}] = 1 - \alpha$$

so that

$$P[1/Y < 1/F_{r,s,1-\alpha}] = 1 - \alpha$$

But, since  $1/Y \sim F_{s,r}$ , by the definition of upper cutoff, the last line says that

$$1/F_{r,s,1-\alpha} = F_{s,r, \alpha}$$