

(Note pdf's are 0 off of the indicated sets.)

I. IMPORTANT DISCRETE DISTRIBUTIONS.

1) **Name :** Binomial **Shorthand :** $X \sim B(n,p)$

$$\text{pdf: } f(x) = \binom{n}{x} p^x q^{n-x}, x \in \{0,1,\dots,n\}$$

$$\mu = \text{mean} = np, \text{ variance} = \sigma^2 = npq$$

2) **Name :** Geometric **Shorthand :** $X \sim \text{Ge}(p)$

$$\text{pdf: } f(x) = q^{x-1} p, x \in \{1,2,\dots\}$$

$$\mu = \frac{1}{p} \quad \sigma^2 = q / p^2$$

3) **Name :** Hypergeometric **Shorthand :** $X \sim H(N,D,n)$

$$\text{pdf: } f(x) = \frac{\binom{D}{x} \binom{N-D}{n-x}}{\binom{N}{n}}$$

$$\mu = n \frac{D}{N} \quad \sigma^2 = n \frac{D}{N} \left(1 - \frac{D}{N}\right) \left(\frac{N-n}{N-1}\right)$$

4) **Name :** Poisson **Shorthand :** $X \sim P(\mu)$

$$\text{pdf: } f(x) = \frac{\mu^x e^{-\mu}}{x!}, x \in \{0,1,\dots\}$$

$$\mu = \mu \quad \sigma^2 = \mu$$

II. IMPORTANT CONTINUOUS DISTRIBUTIONS

1) **Name:** Normal **Shorthand:** $X \sim N(\mu, \sigma^2)$

$$\text{pdf: } f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$\mu = \mu \quad \sigma^2 = \sigma^2$$

2) **Name:** Gamma **Shorthand :** $X \sim G(\alpha, \lambda)$

$$\text{pdf: } f(x) = \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}, x > 0$$

$$\mu = \frac{1}{\lambda} \quad \sigma^2 = \frac{1}{\lambda^2}$$

3) **Name:** Beta **Shorthand :** $X \sim \text{Be}(\alpha, \beta)$

$$\text{pdf: } f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}, 0 < x < 1$$

$$\mu = \frac{\alpha}{\alpha + \beta} \quad \sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

4) **Name:** Weibull **Shorthand :** $X \sim \text{We}(\alpha, \beta)$

$$\text{pdf: } f(x) = \alpha \beta x^{\beta-1} \exp(-\beta x^\alpha)$$

$$\mu = \frac{1}{\beta} \Gamma(1 + 1/\alpha) \quad \sigma^2 = \frac{1}{\beta^2} \left[\Gamma(1 + 2/\alpha) - \mu^2\right]$$

Notes on Linear Combinations of Random Variables

I. Summary of Reproductive Properties

X_1, X_2, \dots, X_k are independent random variables.

$$S = \sum_{i=1}^k X_i.$$

If

then

$$X_i \sim B(n_i, p) \quad S \sim B\left(\sum_{i=1}^k n_i, p\right)$$

$$X_i \sim P(\lambda_i) \quad S \sim P\left(\sum_{i=1}^k \lambda_i\right)$$

$$X_i \sim G(\lambda_i, \mu_i) \quad S \sim G\left(\sum_{i=1}^k \lambda_i, \mu_i\right)$$

$$X_i \sim N(\mu_i, \sigma_i^2) \quad S \sim N\left(\sum_{i=1}^k \mu_i, \sum_{i=1}^k \sigma_i^2\right)$$

II. Summary of expectation information for linear combinations of random variables.

X_1, X_2, \dots, X_k are random variables with means $E[X_i] = \mu_i$, variances $\text{Var}(X_i) = \sigma_i^2$, and covariances $\sigma_{ij} = \text{Cov}(X_i, X_j)$.

If

$$Y = \sum_{i=1}^n a_i X_i$$

then

$$\mu_Y = E(Y) = \sum_{i=1}^n a_i \mu_i$$

and

$$\sigma_Y^2 = \sum_{i=1}^n a_i^2 \sigma_i^2 + \sum_{i \neq j} a_i a_j \sigma_{ij}.$$

If X and Y are independent random variables then $\text{Cov}(X, Y) = 0$.