

Example.. Consider the following data in which there are two factors. See example 11.7, page 452, Devore 5th ed. for a complete explanation of this study conducted to assess the effects of density of planting and variety type on the yield of Tomato plants. The analysis below is conducted under the assumption that the yields satisfy

$$Y_{ijk} = \mu_{ij} + \epsilon_{ijk},$$

where ϵ_{ijk} are iid $N(0, \sigma^2)$.

- (a) Does the data present significant evidence at $\alpha = 0.05$ that variety and/or density have any effect on mean yield?

Solution:

- hypothesis - Test H1: $\mu_{ij} = \text{constant}$ vs. K1: not H1
- Decision rule - Reject H1 if the overall $F = MS_{Model} / MS_{Error}$ exceeds $F_{IJ-1, IJ(K-1), \alpha}$, where $I = 3, J = 4, K = 3$.
Alternative statement of the decision rule in terms of p-values - Reject H1 if the p-value for the overall $F = MS_{Model} / MS_{Error}$ is less than $\alpha = 0.05$.
- Decision - Since $F = 24.22 > F_{11, 24, 0.05} = 2.21$, reject H1 and conclude that the data presents significant evidence of some relationship between the variables variety, density of planting, and yield.
Alternatively in terms of p-values - The p-value for the overall $F = MS_{Model} / MS_{Error}$ is 0.0001 which is less than $\alpha = 0.05$ so reject H1.

Below, we have expressed $\mu_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij}$, where

$$\sum_i \alpha_i = \sum_j \beta_j = \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0.$$

- (b) Does the data present significant evidence of an interaction between variety and planting density at $\alpha = .05$?

- hypothesis - Test H2: $\gamma_{ij} = 0$ all i, j vs. K2: not H2
- Decision rule - Reject H2 if the $F = MS_{Interaction} / MS_{Error}$ exceeds $F_{(I-1)(J-1), IJ(K-1), \alpha}$.
Alternative statement of the decision rule in terms of p-values - Reject H2 if the p-value for the $F = MS_{Interaction} / MS_{Error}$ is less than $\alpha = 0.05$.
- Decision - Since $F = 0.84 < F_{6, 24, 0.05} = 2.51$, do not reject H2 and conclude that the data does not present significant evidence of an interaction between the variables variety and density of planting.
Alternatively in terms of p-values - The p-value for the $F = MS_{Interaction} / MS_{Error}$ is 0.5484 which is greater than $\alpha = 0.05$ so do not reject H2.

- (c) Does the data present significant evidence at $\alpha = .05$ that variety has an effect on mean yield?

- hypothesis - Test H3: $\mu_i = 0, i=1, 2, 3$ vs. K3: not H3
- Decision rule - Reject H3 if the $F = MS_{Variety} / MS_{Error}$ exceeds $F_{I-1, IJ(K-1), \alpha}$.

Alternative statement of the decision rule in terms of p-values - Reject H3 if the p-value for the $F = MS_{Variety} / MS_{Error}$ is less than $= 0.05$.

• Decision - Since $F = 103.34 > F_{2,24,0.05} = 3.40$, reject H3 and conclude that the data presents significant evidence at $\alpha = 0.05$ that variety has an effect on mean yield.

Alternatively in terms of p-values - The p-value for the $F = MS_{Variety} / MS_{Error}$ is 0.0001 which is less than $\alpha = 0.05$ so reject H3.

(c) Does the data present significant evidence at $\alpha = .05$ that planting density has an effect on mean yield?

• hypothesis - Test H4: $\mu_j = 0, j = 1, \dots, 4$ vs. K4: not H4

• Decision rule - Reject H4 if the $F = MS_{Density} / MS_{Error}$ exceeds $F_{J-1, IJ(K-1), \alpha}$.

Alternative statement of the decision rule in terms of p-values - Reject H4 if the p-value for the $F = MS_{Density} / MS_{Error}$ is less than $= 0.05$.

• Decision - Since $F = 18.23 > F_{3,24,0.05} = 3.01$, reject H4 and conclude that the data presents significant evidence at $\alpha = 0.05$ that planting density has an effect on mean yield.

Alternatively in terms of p-values - The p-value for the $F = MS_{Density} / MS_{Error}$ is 0.0001 which is less than $\alpha = 0.05$ so reject H4.

yld	dens	var
10.5	10	h
9.2	10	h
7.9	10	h
12.8	20	h
11.2	20	h
13.3	20	h
12.1	30	h
12.6	30	h
14	30	h
10.8	40	h
9.1	40	h
12.5	40	h
8.1	10	ife
8.6	10	ife
10.1	10	ife
12.7	20	ife
13.7	20	ife
11.5	20	ife
14.4	30	ife
15.4	30	ife
13.7	30	ife
11.3	40	ife
12.5	40	ife
14.5	40	ife
16.1	10	p
15.3	10	p

```
17.5 10 p
16.6 20 p
19.2 20 p
18.5 20 p
20.8 30 p
18 30 p
21 30 p
18.4 40 p
18.9 40 p
17.2 40 p
*****
options ls=75;
options pagesize=50;
title 'DeVore, Example 11.7';
filename fac 'ex11.7.data';
data ylds;
infile fac;
input y d v$;
proc anova;
class d v;
model y = d v d*v;
```

Analysis of Variance Procedure
 Class Level Information

Class	Levels	Values
D	4	10 20 30 40
V	3	h i f e p

Number of observations in data set = 36

Analysis of Variance Procedure

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	422.315556	38.392323	24.22	0.0001
Error	24	38.040000	1.585000		
Corrected Total	35	460.355556			
	R-Square	C.V.	Root MSE		Y Mean
	0.917368	9.064568	1.25897		13.8889

Source	DF	Anova SS	Mean Square	F Value	Pr > F
D	3	86.686667	28.895556	18.23	0.0001
V	2	327.597222	163.798611	103.34	0.0001
D*V	6	8.031667	1.338611	0.84	0.5484