We present in the next example an analysis based on the use of our hypothesis testing summary sheet and an alternative analysis based upon a SAS printout.

Consider the following two-sample problem. Recovery times for two surgical procedures, 1 and 2 are investigated. The data are the observed recovery times and may be considered as independent random samples from two normal populations.

1 8.161
1 7.062
1 8.075
1 7.495
1 7.321
1 6.877
1 7.857
1 8.191
1 8.644
1 7.917
1 9.253
1 6.785
2 10.662
2 10.537
2 9.7035
2 10.029
2 9.0677
2 9.0919
2 9.6989
2 10.358
2 8.2318

(Treatment 1 is also labeled x below and treatment 2 is labeled y.)

Is there significant evidence at $\alpha = 0.05$ that the mean times to recovery for the two procedures differ?

I. - Traditional solution.

(1) Hypothesis of interest : $H_0 : \mu_1 = \mu_2$ vs. $H_A : \mu_1 \neq \mu_2$

Since the sample sizes are small and the validity of the t-test which is appropriate for testing our hypothesis depends on equality of the variances of the two populations, we will need to execute a preliminary test of

(1) Hypothesis' : $H'_0 : \sigma_1 = \sigma_2$ vs. $H'_A : \sigma_1 \neq \sigma_2$

and we shall make this test at level $\alpha' = 0.10$.

(2) Decision rule: Reject H'_0 if $\tau < F_{n(1)-1,n(2)-1,1-\alpha'/2}$ or if $\tau > F_{n(1)-1,n(2)-1,\alpha'/2}$, where $\tau = s_1^2/s_2^2$.

(3) Decision: Since $\tau = 0.848$, $F_{11,8,.05} \sim 3.31$ and $F_{11,8,.95} = 1/2.95 \sim .338$ do not reject H'_0 and proceed as if the variances were equal.

(2) Decision Rule : Reject H_0 if $|\tau| > t_{n(1)+n(2)-2,.025}$, where

$$\tau = \frac{\bar{x} - \bar{y}}{s_p\sqrt{1/n + 1/m}}$$

and $s_p^2 = \dfrac{(n-1)s_X^2 + (m-1)s_Y^2}{m+n-2}$.

(3) Decision : Since $t_{19,.025} = 2.093$ and $|\tau| = 5.7 > 2.093$, reject H_0 and conclude that the data does present significant evidence that the mean recovery times for the two procedures differ.

II - Solution using a SAS printout.

The following output of the SAS procedure PROC TTEST can be used to test the above. The output is :

```
                             Two Sample example                                1
                                          12:34 Monday, August 25, 1997

                               TTEST PROCEDURE

Variable: RTM

T       N           Mean        Std Dev      Std Error        Minimum        Maximum
-------------------------------------------------------------------------------------
1      12     7.80316667     0.73536965     0.21228293     6.78500000     9.25300000
2       9     9.70886667     0.79778788     0.26592929     8.23180000    10.66200000

Variances        T         DF     Prob>|T|
---------------------------------------
Unequal     -5.6006      16.6       0.0001
Equal       -5.6695      19.0       0.0000
```

```
For H0: Variances are equal, F' = 1.18    DF = (8,11)    Prob>F' = 0.7815
```

and the program that yields this output is

```
options ls=79;
options pagesize=55;
TITLE'Two Sample example';
filename rec 'twosam';
DATA rt;
infile rec;
INPUT t rtm;
PROC TTEST;
CLASS t;
VAR rtm;
```

We begin, as usual, with a statement of the hypothesis and alternative to be tested. The steps that follow are a bit different.

(1) Hypothesis of interest : $H_0 : \mu_1 = \mu_2$ vs. $H_A : \mu_1 \neq \mu_2$.

Since the sample sizes are small and the validity of the t-test appropriate for testing our hypothesis depends on equality of the variances of the two populations, we will need to execute a preliminary test of

(1) Hypothesis' : $H'_0 : \sigma_1 = \sigma_2$ vs. $H'_A : \sigma_1 \neq \sigma_2$

and will make this test at level $\alpha' = 0.10$.

(2) Decision rule: Reject H'_0 if the p-value for the variance test is less than 0.10.

(3) Decision: Sincethe p-value for the variance test is .7815, do not reject H'_0 and proceed as if the ariances were equal.

(2) Decision rule : After failing to reject the hypothesis of equality of variances, reject H_0 if the p-value of the t statistic for this data under two-sided hypothesis is less than 0.05.

(3) The appropriate p-value is given on the SAS printout for this case and is less than 0.0000 (it is zero to four places), which is less than 0.05, so reject H_0 and conclude that the data does present significant evidence that the mean recovery times for the two procedures differ.

By the way, I generated this data; the x's are N(8,1) and y's are N(9.5,1).