Some Testing Examples

Weds. Aug 20.

Example 1.

The following observations are a random sample from a normal distribution. Does the data present significant evidence at $\alpha = 0.05$ that the mean of the distribution is greater than 13 ?

17.116458	16.827330
13.478285	18.959281
14.545863	15.634553
19.269682	14.908234
13.965917	13.902766
23.529665	16.101712

Solution:

1. *Hypothesis* : Test H_0: $\mu \le 13$ vs H_a: $\mu > 13$
2. *Decision rule* : Reject H_0 if $\tau > t_{n-1,\alpha}$, where

$$\tau = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

and $\mu_0 = 13$.

3. *Decision* : The sample average is

$$\bar{x} = 16.520,$$

the sample standard deviation is

$$s = 2.91$$

and the test statistic is

$$\tau = 4.190.$$

Since $t_{11,0.05} = 1.796$ and 4.190 > 1.796, reject Ho and conclude there is significant evidence the mean is greater than 13.

By the way, I generated the data using a random number generator and the true data is N(15,9).

Example 2.

The data below is a random sample from a normal population. Does the data present significant evidence at $\alpha = 0.05$ that the variance of the population exceeds 10?

18.166203
20.294532
12.654959
22.019530
15.990669
22.580638
13.657798
25.453474
20.995765
21.765668

Solution:

1. *Hypothesis* : Ho : $\sigma^2 \leq 10$ vs Ha: $\sigma^2 > 10$
2. *Decision rule* : Reject H_o if $\tau > \chi^2_{n-1,\alpha}$, where

$$\tau = \frac{(n-1)s^2}{\sigma_0^2}$$

and $\sigma_0^2 = 10$.

3. *Decision* : The sample variance is

$$s^2 = 17.13$$

and the test statistic is

$$\tau = 15.416.$$

Since $\chi^2_{9,0.05} = 16.92$ and $15.416 < 16.92$, do not reject Ho and conclude there is not significant evidence the variance exceeds 10.

By the way, I generated the data using a random number generator and the true data is N(20,16).