

MA 3215
Prehistoric Exam 2

Name _____

You may use your crib sheet, your calculator, and the tables provided.
Please use no other outside source of information. **Show your work.**

1) Calls arrive according to a Poisson process $\{ X(t) : t \geq 0 \}$, t in minutes,
with arrival rate $\lambda = 2$ per minute.

a) What is the expected number of arrivals in the first 5 minutes ?

b) What is the probability that more than 15 calls arrive in the first 5
minutes ?

c) What is the expected time of arrival of the 7th call ?

d) What is the probability that the time of arrival of the 7th call is later
than 4 minutes ?

2) The random variable Z has a standard normal distribution; $Z \sim N(0,1)$.

a) Write out the density function of Z .

b) What is the moment generating function of Z ?

c) $E(Z) =$ _____

d) $\text{Var}(Z) =$ _____

e) What is the moment generating function of $Y = 3Z + 2$?

f) Write out the probability density function of $Y = 3Z+2$.

g) $P[Z \leq 1.94] =$ _____

h) $P[Z \leq -0.35] =$ _____

i) $P [-1.23 \leq Z \leq 2.54] =$ _____

j) $P[| Z | \leq 1] =$ _____

3) The random variable X has a normal distribution with mean $\mu = 3$ and variance $\sigma^2 = 16$; $X \sim N(3,16)$.

a) $P[X \leq 7.25] = \underline{\hspace{2cm}}$

b) If $P[X \leq c] = 0.063$ then $c = \underline{\hspace{2cm}}$

c) $E\left[\frac{X - 3}{4} \right] = \underline{\hspace{2cm}}$

d) $E[(2X + 1)^2] = \underline{\hspace{2cm}}$

4) The random variable Y has probability density function

$$f(y) = \begin{cases} 3y^2 & \text{if } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases} .$$

a) $P[Y \leq .5] =$ _____

b) $E[Y] =$ _____

c) Give a closed form expression for the moment generating function of Y.

d) Give the formula for the cdf $F(y)$.

e) Find $F(0.45)$.

f) Find the formula for the inverse $F^{-1}(u)$ for $0 < u < 1$.

g) Describe, in complete detail, how to simulate an observation of Y .