

Ma3215 Exam 2 Summer 2001

Name _____
 Table # _____

You may use your two crib sheets, your calculator, and the tables I supply. Use no other outside source of information. **Show your work.**

In working the problems on this exam you may use the following

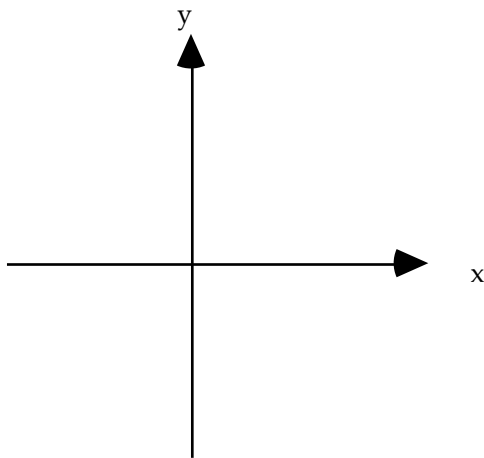
FACT: If n is an integer greater than 0 then $\int_0^{\infty} x^{n-1} e^{-x} dx = (n-1)!$ (note $0! = 1$).

1. In a communications system, time X between incoming queries is exponentially distributed. Once the query has been received, it takes a random amount of time to process it. The total time used by the system to wait for one query and to process it is Y . Assuming the random variables X and Y have joint density

$$f_{XY}(x, y) = \begin{cases} e^{-y} & y \geq x \geq 0 \\ 0 & \text{otherwise} \end{cases},$$

answer the following questions.

(a) Sketch the outcome space for (X, Y) .



(b) Find the marginal pdf f_Y of Y .

(c) Are X and Y independent? Why?

(d) Find $E[Y]$.

(e) Find $\text{Var}(Y)$.

(f) Find $E[XY]$.

(g) Find for $t > 0$, $P[Y - X \leq t] =$

2. If X and Y are random variables with $E[X] = 2$, $E[X^2] = 6$, $E[Y] = 10$, $\text{Var}(Y) = 10$, and $\rho_{XY} = .2$, then

(a) $\text{Var}(X) =$

(b) $\rho_Y =$

(c) $\rho_{XY} =$

(d) $E[2X - 3Y] =$

(e) $\text{Var}(2X - 3Y) =$

(f) $P[X > Y] =$ (check one)

(i) $1/5$

(ii) $2/3$

(iii) it is not possible to tell

(iv) none of these

3. The random time T until failure of a router has probability density

$$f_T(t) = \begin{cases} (1+t)^{-2} & \text{for } t \geq 0 \\ 0 & \text{otherwise} \end{cases} .$$

You are going to sell warranties on new routers. The price of a router is R and if it fails at time $t < g$ you pay $R(1-t/g)$ and if it lives past the warranted lifetime g , you pay nothing. Assuming you are in the warranty business to make money, what is the smallest amount S you can charge per warranty and still avoid losing money on the average if $R = 5$ and $g = 1$?