

Ma 4216
Test 2
sample

Name _____

You may use your calculator, your crib sheet, and the tables supplied. Please use no other outside source of information. **Show your work.**

1. Let p denote the proportion of letters mailed in the Netherlands that are delivered the next day. In a random sample of 100 letters, 73 were delivered the next day. Does this data present significant evidence at $\alpha = 0.05$ that p differs from 0.75 ?

2. Let X_1, \dots, X_n be a random sample from $N(2, \sigma^2)$.

2a) Show that $C = \{ (x_1, \dots, x_n) : \sum_{i=1}^n (x_i - 2)^2 \leq c \}$ is a best critical region for testing

$H_0: \sigma^2 = 4$ against $H_A: \sigma^2 = 9$.

2b) Find c if the best test is to have size $\alpha = 0.05$.

3. A random sample of 8 E-glass fiber test specimens of a certain type yielded a sample average interfacial shear yield stress of 32.3 and a sample standard deviation of 2.1. Assuming that interfacial shear yield stress is normally distributed is there significant evidence at $\alpha = 0.01$ that the mean interfacial shear yield stress exceeds 30 ?

4. A pollution control inspector suspected that a river community was releasing amounts of semitreated sewage into a river. To check his theory, he drew eight randomly selected specimens of river water at a location above the town and another eight below. The dissolved oxygen readings, in parts per million are as follows:

ABOVE TOWN	3.68	5.96	4.26	4.94	5.27	6.01	4.87	5.82
BELOW TOWN	2.54	2.46	4.67	2.69	3.07	4.27	4.00	2.60

The accompanying printout entitled TT4216 contains the results of a SAS analysis with LOC 1 above town and LOC 2 below town. (Assume the data is normally distributed.)

- (a) Give a point estimate of the mean dissolved oxygen content above town.
- (b) Give a point estimate of the standard deviation of the dissolved oxygen content above town.

(c) Test the hypothesis

$$H_0: \frac{\sigma_A^2}{\sigma_B^2} = \frac{\sigma_A^2}{\sigma_B^2} \text{ against the alternative } H_A: \frac{\sigma_A^2}{\sigma_B^2} \neq \frac{\sigma_A^2}{\sigma_B^2}$$

at $\alpha = 0.05$.

(d) Do the data present significant evidence at $\alpha = 0.01$ to indicate that the mean oxygen content below town is less than the mean oxygen content above town?

5. Let X_1, \dots, X_n be a random sample from $N(\mu, 9)$.

a) Write out the probability density for X_1 .

b) Write out the joint probability density $f(x | \mu)$ of $X = (X_1, \dots, X_n)$.

c) Derive the maximum likelihood estimator of μ .

d) Write out the likelihood ratio $\lambda(x)$ for testing

$$H_0: \mu = 10 \text{ against } H_A: \mu \neq 10.$$

e) Prove that for any real number t ,

$$\sum_{i=1}^n (x_i - t)^2 = \sum_{i=1}^n (x_i - \bar{x})^2 + n(\bar{x} - t)^2.$$

f) Derive the form of the critical region of the likelihood ratio test of H_0 against H_A .

g) Give the likelihood ratio test of size $\alpha = 0.01$.

XX

```
options ls=79;
options pagesize=55;
TITLE 'TT4216';
DATA DVS;
INPUT LOC;
DO I=1 TO 8;
INPUT DSO @@;
OUTPUT;
END;
CARDS;
1
3.68 5.96 4.26 4.94 5.27 6.01 4.87 5.82
2
2.54 2.46 4.67 2.69 3.07 4.27 4.00 2.60
;
PROC TTEST;
CLASS LOC;
VAR DSO;
```

TT4216

19, 1998

TTEST PROCEDURE

Variable: DSO

LOC	N	Mean	Std Dev	Std Error	Minimum
Maximum					

1	8	5.10125000	0.83798121	0.29627110	3.68000000
6.01000000					
2	8	3.28750000	0.88678794	0.31352688	2.46000000
4.67000000					

Variances	T	DF	Prob> T

Unequal	4.2047	14.0	0.0009
Equal	4.2047	14.0	0.0009

For H0: Variances are equal, F' = 1.12 DF = (7,7) Prob>F' = 0.8851