

Social Security Retirement Benefits for Single Retirees Age 62*

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1 Introduction

For each individual younger than 62 covered by Social Security (SS) and eligible for retirement benefits, a decision must be made concerning whether to begin retirement benefits at 62 with reduced payments or to wait until the allotted retirement age and receive full benefits. For the purposes of the calculations presented here it is assumed that the individual has retired already so there will be no additional contributions to their SS. It is also assumed that there is no spouse, that the individual will not work again, and no inflation possibilities or tax consequences are taken into account. Present values at 62 for the two courses of action are calculated at fixed interest rates and their expected values computed according to life tables from 2003 of white males. With these caveats in mind, and to the extent that it is possible to rely on the relative constancy of interest rates in a world in which future interest rates are unknown, the calculations still may be of some use in reaching a decision in this issue.

My particular benefits are $1478 if I take them early and $1960 if I wait till 66. Based on life tables for white males, for all reasonable interest rates from 2% up, the expected present value for my particular benefits are greater by taking early retirement at 62 than those resulting from waiting to take full benefits at age 66. At 7% or greater prevailing interest rates, for these particular benefits the answer is unequivocal; no matter what your race or sex, taking early SS benefits results in higher present values for early retirement. One must take care however since these statements do not necessarily apply to other benefits configurations. One must investigate each case separately.

A calculator has been provided to help in the investigation of the present values under various interest rates and amounts. It applies only to 62 vs 66 years as retirement ages[1]. If information is

*Nothing in this document or its associated calculator constitutes a recommendation of any kind. It is solely for informational purposes and is provided as an aid in the decision making process. Although every attempt has been made to make the presentation accurate, it has not been checked and should not be relied upon without further corroboration from independent sources

[1] It also makes some technical assumptions presented in the body of the writeup.

desired about other alternatives, say 64 vs 66 for example or 62 vs 67, then this calculator does not give that information. Nor does it provide expected value information for any other demographic than white males. The present value curves plotted by the calculator do apply to all demographics, but only for 62 vs 66. Just enter (no dollar signs for the money or percent sign for the interest rates) your reduced monthly benefit for retirement at 62, your full benefit at age 66, and the interest rate at which the computation is to be performed.

2 Present Value

What is the value at time 0 of getting amounts A_i at times t_i, $0 < t_1 < t_2 < \ldots < t_k < T$ and investing them until time T? At the end of T we will have $\sum_{j=1}^{k}(1+\alpha)^{T-t_j}A_j$. Denoting the present value by V, we have $(1+\alpha)^T V = \sum_{j=1}^{k}(1+\alpha)^{T-t_j}A_j$ so

$$V = \sum_{j=1}^{k}(1+\alpha)^{-t_j}A_j.$$

The following example assumes that I will receive \$1478 per month until my death if I elect to start benefits at 62 and I will receive \$1960 per month if I elect to start at my full retirement age of 66.

2.1 At 62, early retirement, lifetime 744+X months

If I am 62 now and elect to take my reduced-benefits retirement immediately then we have for a lifetime $T = 62 \times 12 + X$ in months a present value of

$$\begin{aligned} 1478 \cdot \sum_{j=1}^{X}(1+\gamma/12)^{-j} &= 1478(1+\gamma/12)^{-1}\frac{1-(1+\gamma/12)^{-X}}{1-(1+\gamma/12)^{-1}} \\ &= \frac{12}{\gamma}\cdot 1478 \cdot [1-(1+\gamma/12)^{-X}] \end{aligned}$$

If, for example, I live 3 years and 4 months after starting at 62, then the present value of my retirement (assuming I do not have a wife or any dependents) is, at 5% ($\gamma = 0.05$) and since $X = 40$ months, \$54,352.01. If I had opted to wait until my full retirement age, the present value of my retirement would be 0.

2.2 At 62, SS benefits begin at 66, lifetime 744 +X months

For $X < 49$ the present value is 0. For $X = 48 + Y, Y \geq 1$ we have a present value of

$$\begin{aligned} 1960 \cdot \sum_{j=1}^{Y}(1+\gamma/12)^{-(j+48)} &= 1960(1+\gamma/12)^{-49}\frac{1-(1+\gamma/12)^{-Y}}{1-(1+\gamma/12)^{-1}} \\ &= \frac{12}{\gamma}\cdot 1960(1+\gamma/12)^{-48}\cdot[1-(1+\gamma/12)^{-Y}] \end{aligned}$$

For example, if I live to age 66+3 yrs and 4 months then the present value of the retirement would be (take $Y = 40, \gamma = 0.05$) \$59,036.26. Had I taken the reduced benefit at age 62 and lived to the same point the present value would be (take $X = 88$ above) \$108,697.29. In the figures below can be found plots comparing the two strategies over a range of possible life lengths. Figure 1 is one such, carried out under the assumption of a 5% interest rate. The break-even point there, where the two courses of action yield approximately the same present values, is approximately age 88 years, 9 months. The plot at 2% is found in Figure 2 and break-even for that is around 80 years, 9 months, slightly less than the expected lifetime for a white male, conditional on reaching 62, according to current (2003) life tables. The plot of Figure 3 shows that for interest rates at least 7% early is better uniformly over all ages to 103 and Figure 4 shows that at interest rates of 10% or higher the two can differ substantially over the full spectrum of possible ages.

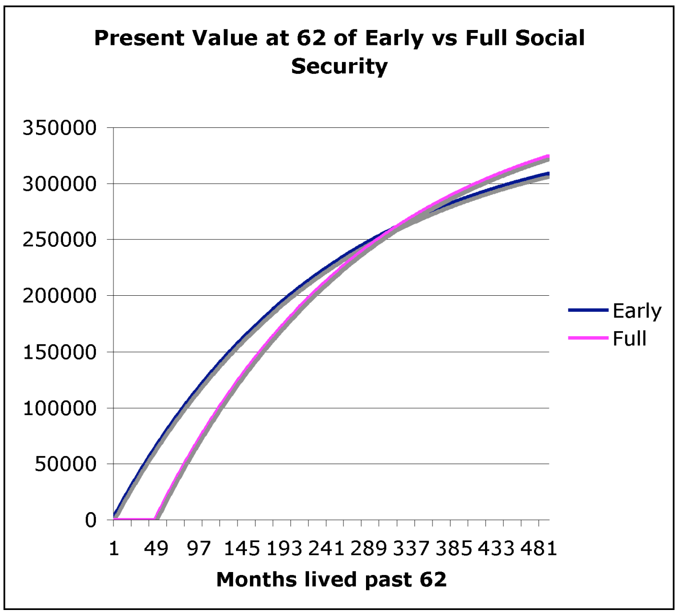

Figure 1: Present value at 5%

3 Using 2003 Mortality Tables

In the previous section we saw a comparison of performance based upon one's life length. In cases in which the present value curves cross, since one does not know in advance one's life length, the graphs do not lead to an unequivocal decision. In this section we shall employ information from mortality tables to aid in our assessment of the relative merits of the two strategies by utilizing the information about probabilities of life lengths.

Calculations are presented below involving the entire probability distribution of life lengths in conjunction with the present values, but first there is a simple approximation which is frequently used based upon *expected lifetimes*. Consulting the life tables from 2003 for white males, one finds at age 62 an expected additional life of approximately 19 years. Then we take $x = 228$ in the

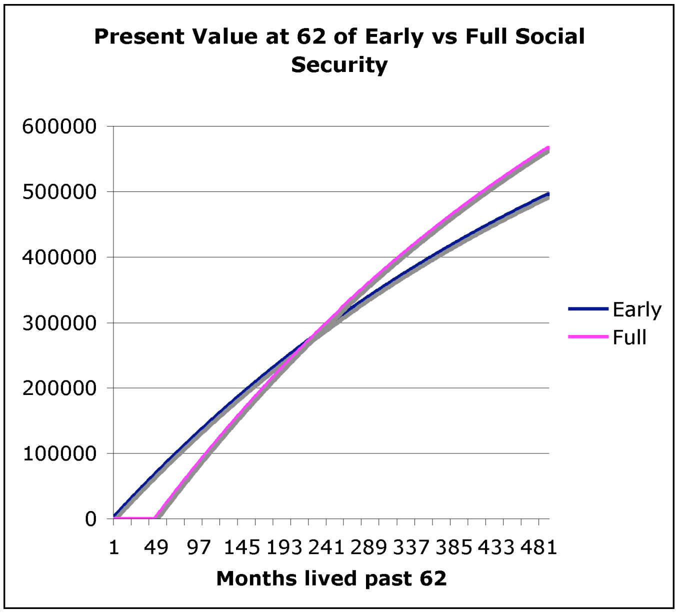

Figure 2: Present value at 2%

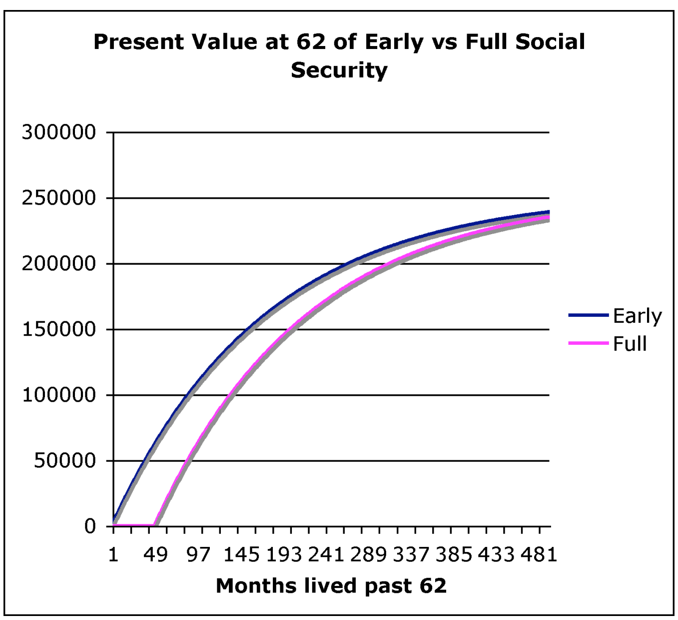

Figure 3: Present value at 7%

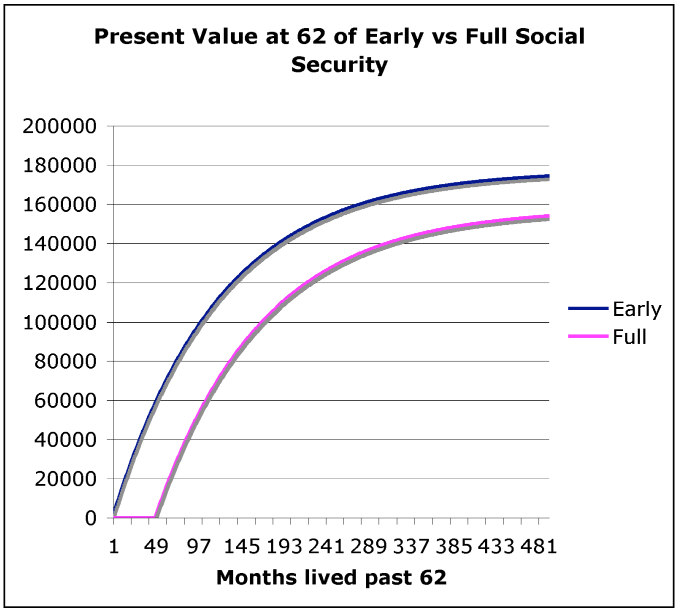

Figure 4: Present value at 10%

Interest rate	Early	Full
2%	280,159.84	281,181.67
5%	217,264.20	203,008.62
6%	200,792.58	182,816.90
7%	186,101.01	164,941.60
10%	150,622.84	122,464.21

Table 1: Present Value at Expected Lifetime Under Varying Interest Rates

reduced-benefits computation and $y = 180$ for the full case. At a 5% interest rate, the expected present value for the early is \$217,264.20, while that for the full is \$203,008.62. In Table 1 can be found the results of similar calculations as a function of interest rates. In all cases except the lowest interest rate listed, the present value at the expected lifetime of a 62 year old white male is greater for the early reduced monthly benefit than for the full benefit.

Proceeding to the more precise calculations utilizing the full life tables, we compute the expected present value under two scenarios: early reduced benefits starting at 62, and full benefits starting at 66.

3.1 Estimating Probabilities

We shall utilize the probability estimates from the NCHS (see http://www.cdc.gov/nchs/hus.htm) statistics. A key quantity they provide is an estimate of the conditional probability q_x of dying between x and $x+1$ given the age x has been attained. For example, based on the white male figures from 2003, at $x = 60$ we have $q_x = 0.011924$. We shall denote by T the lifetime of a randomly selected

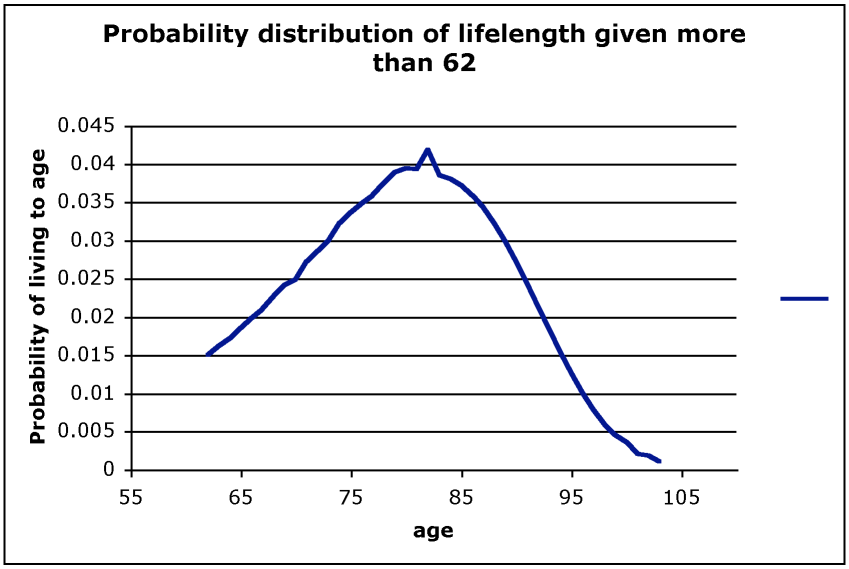

Figure 5: Conditional Distribution of Age of Death for White Males - 2003

individual from this population. Thus, $P[T = x+1|T > x] = q_x$. From these we can compute various quantities. For example, $P[T > x+1|T > x] = 1-q_x$ and since $\{T > x+1\} = \{T > x\} \cap \{T > x+1\}$

$$\begin{aligned}
P[T > x+1] &= P[T > x+1|T > x]P[T > x] \\
&= (1-q_x)P[T > x] \\
&= (1-q_x)(1-q_{x-1})P[T > x-1] \\
&\vdots \\
&= (1-q_x)(1-q_{x-1})\ldots(1-q_0).
\end{aligned}$$

Similarly, $P[T = x+1] = q_x(1-q_{x-1})\ldots(1-q_0)$ and for y an integer, since $P[T > x+y+1|T > x] = \frac{P[T>x+y+1]}{P[T>x]}$

$$P[T > x+y+1|T > x] = (1-q_{x+y})(1-q_{x+y-1})\ldots(1-q_x).$$

Similarly, for $y \geq 1$

$$P[T = x+y+1|T > x] = q_{x+y}(1-q_{x+y-1})\ldots(1-q_x).$$

For $y = 0$ we have $P[T > x+y|T > x] = 1$ and for $y \geq 1$

$$\begin{aligned}
P[T > x+y|T > x] - P[T > x+y+1|T > x] &= (1-q_{x+y-1})\ldots(1-q_x)[1-(1-q_{x+y})] \\
&= P[T = x+y+1|T > x].
\end{aligned}$$

For $y = 0$ one has $P[T = x+y+1|T > x] = q_x$. The conditional probability distribution of lifelength given it is greater than 62, $P[T = 62+x|T > 62]$ is plotted in Figure 5.

Interest rate	Early	Full
2%	260,565.36	258,200.74
5%	196,695.12	178,504.55
6%	180,879.15	159,110.03
7%	167,070.87	142,336.00
10%	134,792.44	103,904.02

Table 2: Expected Present Value Under Varying Interest Rates

3.2 Expected Value

Since our life distributions are only by year we must make an approximation of some kind to assign probabilities to the months. We choose to spread the probability estimated from the mortality tables uniformly over the 12 months for each year and compute from that.

Utilizing the estimated probabilities from the life tables one has the expected present values presented in Table 2. It is interesting to note that the approximation based on expected lifetime considerably overestimates the expected present value.